Worthwhile CAS Calculator Use in This Year's 2nd Methods Exam

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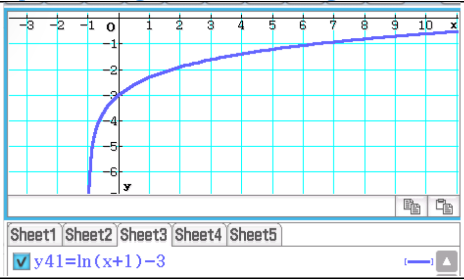
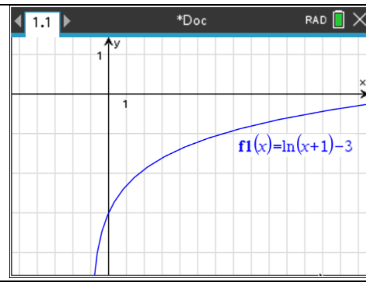
Multiple Choice analysis:

Question 1 (A)

The asymptote(s) of the graph of $y = \log_e(x+1) - 3$ are

Graphical image can be used to give an estimate.





Question 2 (D)

A function $g: R \to R$ has the derivative $g'(x) = x^3 - x$.

Given that $g(0) = 5$, the value of $g(2)$ is

Possible advantage to Specialist students

$\int_0^2 (x^3 - x)\,dx + 5$ 7

$\int_0^2 (x^3 - x)\,dx + 5$ 7

Question 3 (C)

A discrete random variable X is defined using the probability distribution below, where k is a positive real number.

x	0	1	2	3	4
$\Pr(X = x)$	$2k$	$3k$	$5k$	$3k$	$2k$

Find $\Pr(X < 4 \mid X > 1)$

Best done from the table. Scientific calculator would suffice.

$k = \frac{1}{15}$; $\frac{8}{10} = \frac{4}{5}$

Question 4 (B)

If $\int_a^b f(x)dx = -5$ and $\int_a^c f(x)dx = 3$, where $a < b < c$, then $\int_b^c 2f(x)dx$ is equal to

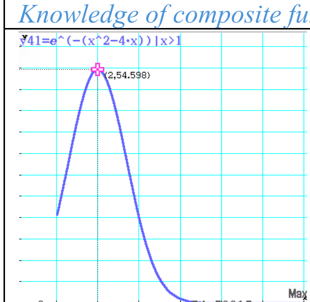
Best done by hand or image.

Question 5 (D)

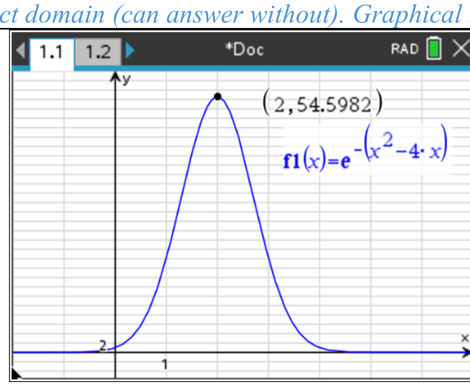
Consider the functions $f: (1, \infty) \to R, f(x) = x^2 - 4x$ and $g: R \to R, g(x) = e^{-x}$.

The range of the composite function $g(f(x))$ is

Knowledge of composite functions helps to formulate the correct domain (can answer without). Graphical image helps.



$\text{fMax}\left(e^{-(x^2-4\cdot x)}, x, 1, \infty\right)$ $\{\text{MaxValue}=e^4, x=2\}$

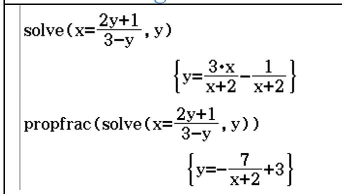


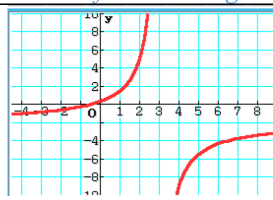
Question 6 (B)

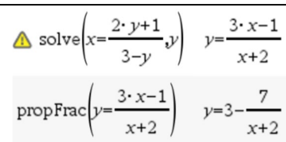
Consider the function $f(x)=\dfrac{2x+1}{3-x}$ with domain $x \in R\backslash\{3\}$.

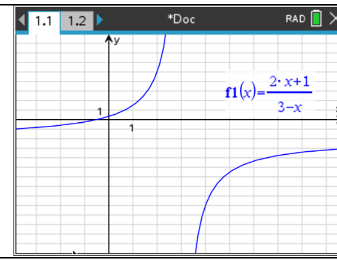
The inverse of f is

Domain/Range considerations could be from an image.





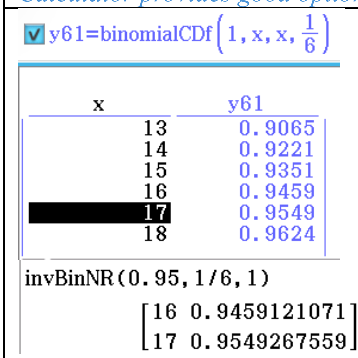


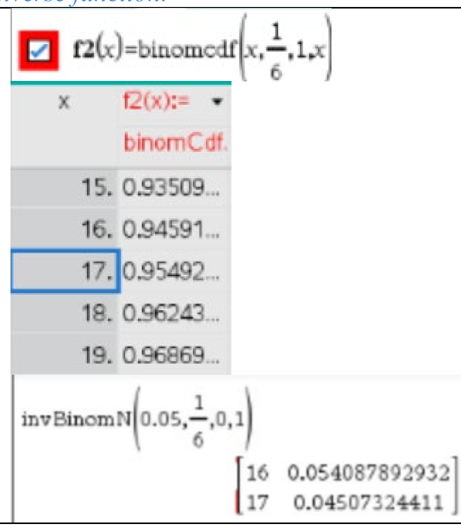


Question 7 (C)

A fair six-sided die is repeatedly rolled. What is the minimum number of rolls required so that the probability of rolling a six at least once is greater than 0.95?

Calculator provides good options either via the table or inverse function.





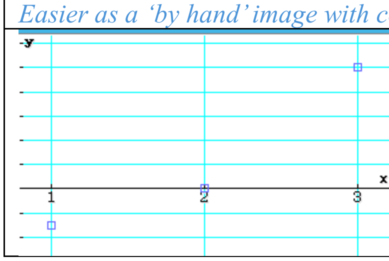
Question 8 (A)

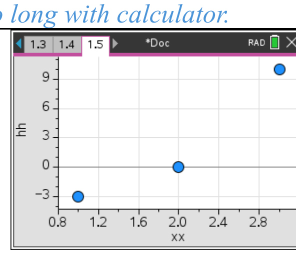
Some values of the functions $f: R \to R$ and $g: R \to R$ are shown below.

x	1	2	3
$f(x)$	0	4	5
$g(x)$	3	4	-5

The graph of the function $h(x)=f(x)-g(x)$ must have an x-intercept at

Easier as a 'by hand' image with coordinates plotted. Too long with calculator.





Question 9 (C)

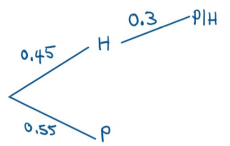
At a Year 12 formal, 45% of the students travelled to the event in a hired limousine, while the remaining 55% were driven to the event by a parent.

Of the students who travelled in a hired limousine, 30% had a professional photo taken.

Of the students who were driven by a parent, 60% had a professional photo taken.

Given that a student had a professional photo taken, what is the probability that the student travelled to the event in a hired limousine?

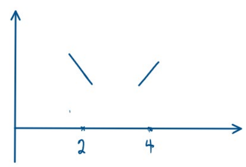
Tree diagram – dependent branches. No calculator.



Question 10 (B)

Suppose a function $f : [0, 5] \to R$ and its derivative $f' : [0, 5] \to R$ are defined and continuous on their domains. If $f'(2) < 0$ and $f'(4) > 0$, which one of these statements must be true?

Hand drawn image provides a good trigger for a solution.



Question 11 (B)

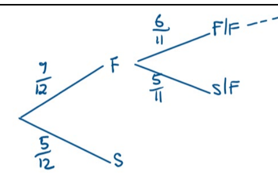
Twelve students sit in a classroom, with seven students in the first row and the other five students in the second row. Three students are chosen randomly from the class.

The probability that exactly two of the three students chosen are in the first row is

Stretch of the study design

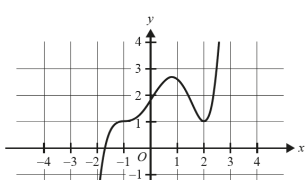
```
hypergeoPDf(2,3,7,12)
                                  0.4772727273
ncr(7,2)ncr(5,1)
   ncr(12,3)
                                            21
                                            44
```

$\dfrac{nCr(7,2) \cdot nCr(5,1)}{nCr(12,3)}$



Question 12 (A)

The graph of $y = f(x)$ is shown below.



Which of the following options best represents the graph of $y = f(2x + 1)$?

Visual application to graphs provided. No CAS assistance.

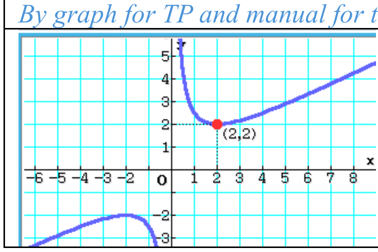
Question 13 (A)

The function $f : (0, \infty) \to R,\ f(x) = \dfrac{x}{2} + \dfrac{2}{x}$ is mapped to the function g with the following sequence of transformations:

1. dilation by a factor of 3 from the y-axis
2. translation by 1 unit in the negative direction of the y-axis.

The function g has a local minimum at the point with the coordinates

By graph for TP and manual for transformations OR function with transformations integrated



define $f(x) = \dfrac{x}{2} + \dfrac{2}{x}$ — done

$\text{fMin}\left(f\left(\dfrac{\frac{x}{3}}{2} + \dfrac{2}{\frac{x}{3}}\right) - 1, x, 0.1, \infty\right)$ — {MinValue=1, x=6}

$\text{fMin}\left(f\left(\dfrac{\frac{x}{3}}{2} + \dfrac{2}{\frac{x}{3}}\right) - 1, x, 0.1, \infty\right)$ — $x=6$

$f\left(\dfrac{\frac{x}{3}}{2} + \dfrac{2}{\frac{x}{3}}\right) - 1 \mid x=6$ — 1

Question 14 (B)

Let h be the probability density function for a continuous random variable X, where

$$h(x)=\begin{cases}\frac{x}{6}+k & -3\le x<0\\ -\frac{x}{2}+k & 0\le x\le 1\\ 0 & \text{elsewhere}\end{cases}$$

and k is a positive real number.

The value of $\Pr(X<0.5)$ is

Separate integral calculation OR piecewise

define $h(x)=\int_{-3}^{0}\frac{x}{6}+k\,dx+\int_{0}^{1}-\frac{x}{2}+k\,dx$ done solve(h(x)=1,k) $\left\{k=\frac{1}{2}\right\}$ $\int_{-3}^{0}\frac{x}{6}+k\,dx+\int_{0}^{\frac{1}{2}}-\frac{x}{2}+k\,dx \mid k=\frac{1}{2}$ $\frac{15}{16}$	Define $h(x)=\begin{cases}\frac{x}{6}+k, & -3\le x<0\\ \frac{-x}{2}+k, & 0\le x\le 1\end{cases}$ Done solve$\left(\int_{-3}^{1}h(x)\,dx=1,k\right)$ $k=\frac{1}{2}$ $\int_{-3}^{\frac{1}{2}}h(x)\,dx\mid k=\frac{1}{2}$ $\frac{15}{16}$

Question 15 (A)

The points of inflection of the graph of $y=2-\tan\left(\pi\left(x-\frac{1}{4}\right)\right)$ are

solve$\left(\frac{d^2}{dx^2}\left(2-\tan\left(\pi\cdot\left(x-\frac{1}{4}\right)\right)\right)=0,x\right)$ $\left\{x=\text{constn}(1)+\frac{1}{4}\right\}$ $2-\tan\left(\pi\cdot\left(x-\frac{1}{4}\right)\right)\mid x=\text{constn}(1)+\frac{1}{4}$ 2	solve$\left(\frac{d^2}{dx^2}\left(2-\tan\left(\pi\cdot\left(x-\frac{1}{4}\right)\right)\right)=0,x\right)$ $x=\frac{4\cdot n1-3}{4}$ $2-\tan\left(\pi\cdot\left(x-\frac{1}{4}\right)\right)\mid x=\frac{4\cdot n1-3}{4}$ 2

Question 16 (D)

Suppose that a differentiable function $f:R\to R$ and its derivative $f':R\to R$ satisfy $f(4)=25$ and $f'(4)=15$.

Determine the gradient of the tangent line to the graph of $y=\sqrt{f(x)}$ at $x=4$.

Could find general derivative from CAS, but must connect notation with given information.

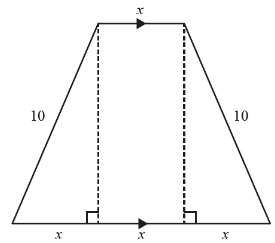
$\frac{d}{dx}\left(\sqrt{f(x)}\right)$ $\frac{\frac{d}{dx}(f(x))}{2\cdot\sqrt{f(x)}}$	$\frac{d}{dx}\left(\sqrt{f(x)}\right)$ $\frac{\frac{d}{dx}(f(x))}{2\cdot\sqrt{f(x)}}$

Question 17 (D)

Consider the algorithm below, which prints the roots of the cubic polynomial $f(x)=x^3-2x^2-9x+18$.

```
define f(x)
    return (x^3 - 2x^2 - 9x + 18)
c ← f(0)
if c < 0 then
```

By hand via a desk check. Too long for CAS programming.

Question 18 (B)	
Find the value of x which maximises the area of the trapezium below. 	
Construction of the area rule (possible difficult) then CAS is great.	
$\text{fMax}\left(\frac{1}{2}\cdot 4\cdot x\cdot\sqrt{10^2-x^2}, x, -\infty, \infty\right)$ $\{\text{MaxValue}=100, x=5\cdot\sqrt{2}\}$	$\text{fMax}\left(\frac{1}{2}\cdot 4\cdot x\cdot\sqrt{10^2-x^2}, x, 1, \infty\right)$ $x=5\cdot\sqrt{2}$

Question 19 (C)	
Consider the normal random variable X that satisfies $\Pr(X < 10) = 0.2$ and $\Pr(X > 18) = 0.2$. The value of $\Pr(X < 12)$ is closest to	
Some time in setting up and the CAS functionality	
$\text{solve}\left(\frac{10-14}{s}=\text{invNormCDf}(0.2, 1, 0), s\right)$ $\{s=4.7527318\}$ $\text{normCDf}(-\infty, 12, 4.7527318, 14)$ 0.3369466892	$\text{solve}\left(\frac{10-14}{s}=\text{invNorm}(0.2,0,1), s\right)$ $s=4.75273$ $\text{normCdf}(-\infty,12,14,4.75273)$ 0.336947

Question 20 (B)
The function $f: R \to R$ has an average value k on the interval $[0, 2]$ and satisfies $f(x) = f(x+2)$ for all $x \in R$. The value of the definite integral $\int_2^6 f(x)dx$ is
By image and transformations

Section B

Question 1a

Consider the function $f : R \to R,\ f(x) = (x+1)(x+a)(x-2)(x-2a)$ where $a \in R$.

a. State, in terms of a where required, the values of x for which $f(x) = 0$.

Standard solving question.

solve((x+1)·(x+a)·(x−2)·(x−2·a)=0, x) {x=−1, x=2, x=−a, x=2·a}	solve((x+1)·(x+a)·(x−2)·(x−2·a)=0,x) x=2·a or x=−a or x=−1 or x=2

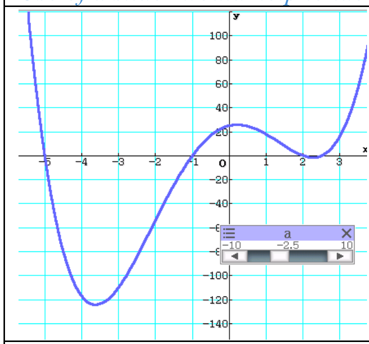
Question 1b

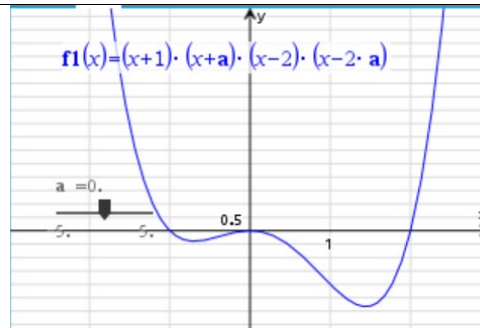
b. Find the values of a for which the graph of $y = f(x)$ has

i. exactly three x-intercepts.

ii. exactly four x-intercepts.

Use of a slider would help to identify.





Question 1ci

c. Let g be the function $g : R \to R,\ g(x) = (x+1)^2(x-2)^2$, which is the function f where $a = 1$.

i. Find $g'(x)$.

Standard derivative

factor($\frac{d}{dx}$((x+1)^2(x−2)^2) 2·(x+1)·(x−2)·(2·x−1)	$\frac{d}{dx}((x+1)^2\cdot(x-2)^2)$ 2·(x−2)·(x+1)·(2·x−1)

Question 1cii

ii. Find the coordinates of the local maximum of g.

Good use of CAS functionality.

fmax((x+1)^2(x−2)^2, x, −1, 2) $\left\{\text{MaxValue}=\frac{81}{16},\ x=\frac{1}{2}\right\}$	fMax((x+1)^2·(x−2)^2,x,−1,2) $\quad x=\frac{1}{2}$ $(x+1)^2\cdot(x-2)^2 \mid x=\frac{1}{2} \quad \frac{81}{16}$

Question 1ciii

iii. Find the values of x for which $g'(x) > 0$.

Good use of CAS functionality.

solve(2·(x+1)·(x−2)·(2·x−1)>0, x) $\left\{-1<x<\frac{1}{2},\ 2<x\right\}$	solve(2·(x−2)·(x+1)·(2·x−1)>0,x) $-1<x<\frac{1}{2}$ or $x>2$

Question 1civ

iv. Consider the two tangent lines to the graph of $y = g(x)$ at the points where

$$x = \frac{-\sqrt{3}+1}{2} \text{ and } x = \frac{\sqrt{3}+1}{2}.$$

Determine the coordinates of the point of intersection of these two tangent lines.

Good use of CAS functionality.

$\text{simplify}\left(y=\text{tanLine}\left((x+1)^2\cdot(x-2)^2, x, \frac{-\sqrt{3}+1}{2}\right)\right)$

$$y=\frac{3\cdot(\sqrt{3}\cdot(4\cdot x-2)+9)}{4}$$

$\text{simplify}\left(y=\text{tanLine}\left((x+1)^2\cdot(x-2)^2, x, \frac{\sqrt{3}+1}{2}\right)\right)$

$$y=\frac{-3\cdot(\sqrt{3}\cdot(4\cdot x-2)-9)}{4}$$

$\begin{cases} y=\frac{3\cdot(\sqrt{3}\cdot(4\cdot x-2)+9)}{4} \\ y=\frac{-3\cdot(\sqrt{3}\cdot(4\cdot x-2)-9)}{4} \end{cases}\Bigg|_{x,y}$

$$\left\{x=\frac{1}{2}, y=\frac{27}{4}\right\}$$

$y=\text{tangentLine}\left((x+1)^2\cdot(x-2)^2, x, \frac{-\sqrt{3}+1}{2}\right)$ $\quad y=3\cdot\sqrt{3}\cdot x-\frac{3\cdot(2\cdot\sqrt{3}-9)}{4}$

$y=\text{tangentLine}\left((x+1)^2\cdot(x-2)^2, x, \frac{\sqrt{3}+1}{2}\right)$ $\quad y=\frac{3\cdot(2\cdot\sqrt{3}+9)}{4}-3\cdot\sqrt{3}\cdot x$

$\text{solve}\left(\begin{cases} y=3\cdot\sqrt{3}\cdot x-\frac{3\cdot(2\cdot\sqrt{3}-9)}{4} \\ y=\frac{3\cdot(2\cdot\sqrt{3}+9)}{4}-3\cdot\sqrt{3}\cdot x \end{cases}, \{x,y\}\right)$ $\quad x=\frac{1}{2} \text{ and } y=\frac{27}{4}$

Question 1di

d. Let g remain as the function $g: R \to R,\ g(x) = (x+1)^2(x-2)^2$, which is the function f where $a = 1$.

Let h be the function $h: R \to R,\ h(x) = (x+1)(x-1)(x+2)(x-2)$, which is the function f where $a = -1$.

i. Using translations only, describe a sequence of transformations of h, for which its image would have a local maximum at the same coordinates as that of g.

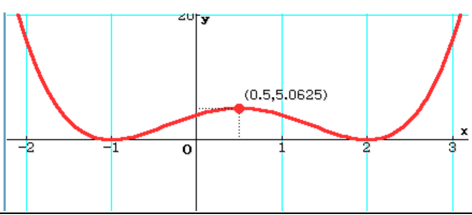
A number of approaches.

$\text{fMax}((x+1)\cdot(x-1)\cdot(x-2)\cdot(x+2), x, -1, 2)$

$$\{\text{MaxValue}=4, x=0\}$$

$\text{fmax}((x+1)^2(x-2)^2, x, -1, 2)$

$$\left\{\text{MaxValue}=\frac{81}{16}, x=\frac{1}{2}\right\}$$



$\text{solve}\left(\frac{d}{dx}\left((x+1)^2\cdot(x-2)^2\right)=0, x\right)$ $\quad x=-1 \text{ or } x=\frac{1}{2} \text{ or } x=2$

$(x+1)^2\cdot(x-2)^2|x=\frac{1}{2}$ $\quad \frac{81}{16}$

$\frac{81}{16}-4$ $\quad \frac{17}{16}$

$\frac{1}{2}-0$ $\quad \frac{1}{2}$

Question 1dii

ii. Using a dilation and translations, describe a different sequence of transformations of h, for which its image would have both local minimums at the same coordinates as that of g.

Good use of CAS functionality.

$\text{fMin}((x+1)\cdot(x-1)\cdot(x-2)\cdot(x+2), x, -2, 2)$

$$\left\{\text{MinValue}=-\frac{9}{4}, x=\frac{-\sqrt{10}}{2}, x=\frac{\sqrt{10}}{2}\right\}$$

$\text{fmin}((x+1)^2(x-2)^2, x, -2, 2)$

$$\{\text{MinValue}=0, x=-1, x=2\}$$

$\text{solve}\left(\left(\frac{\sqrt{10}}{2}-\frac{-\sqrt{10}}{2}\right)\cdot k=2+1, k\right)$

$$\left\{k=\frac{3\cdot\sqrt{10}}{10}\right\}$$

$\text{fMin}((x+1)\cdot(x-1)\cdot(x-2)\cdot(x+2), x, -2, 2)$ $\quad x=\frac{-\sqrt{10}}{2} \text{ or } x=\frac{\sqrt{10}}{2}$

$(x+1)\cdot(x-1)\cdot(x-2)\cdot(x+2)|x=\left\{\frac{-\sqrt{10}}{2}, \frac{\sqrt{10}}{2}\right\}$ $\quad \left\{\frac{-9}{4}, \frac{-9}{4}\right\}$

Question 2a

A model for the temperature in a room, in degrees Celsius, is given by

$$f(t)=\begin{cases}12+30t & 0\le t\le \frac{1}{3}\\ 22 & t>\frac{1}{3}\end{cases}$$

where t represents time in hours after a heater is switched on.

a. Express the derivative $f'(t)$ as a hybrid function.

$\begin{bmatrix}\frac{d}{dt}(12+30t)\\ \frac{d}{dt}(22)\end{bmatrix}$ $\begin{bmatrix}30\\0\end{bmatrix}$	Define $f(t)=\begin{cases}12+30\cdot t, & 0\le t\le \frac{1}{3}\\ 22, & t>\frac{1}{3}\end{cases}$ *Done* $\frac{d}{dt}(f(t))$ $\begin{cases}30, & 0<t<\frac{1}{3}\\ 0, & t>\frac{1}{3}\end{cases}$

Question 2b

b. Find the average rate of change in temperature predicted by the model between $t=0$ and $t=\frac{1}{2}$.

Give your answer in degrees Celsius per hour.

Using pre-defined rules of direct rule

define $f(t)=\begin{cases}12+30t, & 0\le t\le \frac{1}{3}\\ 22, & t>\frac{1}{3}\end{cases}$ done aroc$(0,\frac{1}{2})$ 20	Define $f(t)=\begin{cases}12+30\cdot t, & 0\le t\le \frac{1}{3}\\ 22, & t>\frac{1}{3}\end{cases}$ *Done* $\dfrac{f\left(\frac{1}{2}\right)-f(0)}{\frac{1}{2}-0}$ 20

Question 2ci

c. Another model for the temperature in the room is given by $g(t)=22-10e^{-6t}$, $t\ge 0$.

i. Find the derivative $g'(t)$.

Standard CAS functionality

$\frac{d}{dt}(22-10e^{-6t})$ $60\cdot e^{-6\cdot t}$	$\frac{d}{dt}\left(22-10\cdot e^{-6\cdot t}\right)$ $60\cdot e^{-6\cdot t}$

Question 2cii

ii. Find the value of t for which $g'(t)=10$.

Give your answer correct to three decimal places.

Standard CAS functionality

solve$(60\cdot e^{-6\cdot t}=10, t, 0, -\infty, \infty)$ {t=0.2986265782}	solve$(60\cdot e^{-6\cdot t}=10,t)$ $t=0.298626578205$

Question 2d

d. Find the time $t\in(0,1)$ when the temperatures predicted by the models f and g are equal.

Give your answer correct to two decimal places.

Standard CAS functionality

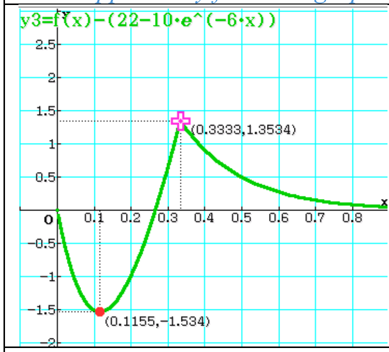
solve$(f(t)=22-10\cdot e^{-6\cdot t}, t)\mid 0<t<1$ {t=0.2656040433}	solve$(f(t)=22-10\cdot e^{-6\cdot t},t)\mid 0<t<1$ t=1.E-38 or t=0.26560404334

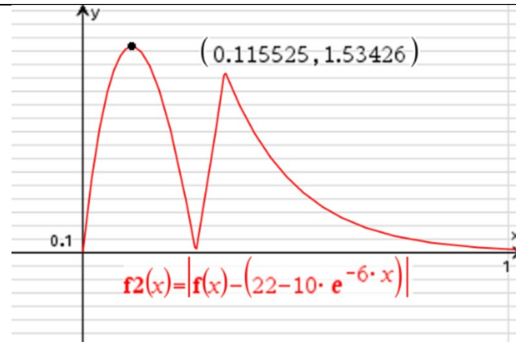
Question 2e

e. Find the time $t \in (0, 1)$ when the difference between the temperatures predicted by the two models is the greatest.

Give your answer correct to two decimal places.

Good opportunity for some graphical work





Question 2fi

f. The amount of power, in kilowatts, used by the heater t hours after it is switched on, can be modelled by the continuous function p, whose graph is shown below.

$$p(t) = \begin{cases} 1.5 & 0 \le t \le 0.4 \\ 0.3 + Ae^{-10t} & t > 0.4 \end{cases}$$

The amount of energy used by the heater, in kilowatt hours, can be estimated by evaluating the area between the graph of $y = p(t)$ and the t-axis.

i. Given that $p(t)$ is continuous for $t \ge 0$, show that $A = 1.2e^4$.

No CAS….show that

Question 2fii

ii. Find how long it takes, after the heater is switched on, until the heater has used 0.5 kilowatt hours of energy.

Give your answer in hours.

Using piecewise definitions

define $p(t)=\begin{cases} 1.5, & 0\le t\le 0.4 \\ 0.3+1.2e^{-10t}, & t>0.4 \end{cases}$

done

solve$\left(\int_0^t p(x)\,dx=0.5, t\right)$

{t=0.3333333333}

Define $p(t)=\begin{cases} 1.5, & 0\le t\le 0.4 \\ 0.3+1.2\cdot e^{-10\cdot t}, & t>0.4 \end{cases}$

Done

solve$\left(\int_0^t p(x)\,dx=0.5,t\right)$

$t=0.333333333333$

Question 2fiii

iii. Find how long it takes, after the heater is switched on, until the heater has used 1 kilowatt hour of energy.

Give your answer in hours, correct to two decimal places.

Integration by two different approaches

solve$\left(\int_0^{0.4} 1.5\,dt+\int_{0.4}^{t} 0.3+1.2\cdot e^{-10\cdot x}\,dx=1, t\right)$

{t=1.726007091}

solve$\left(\int_0^t p(x)\,dx=1,t\right)$ $t=1.72600709055$

Question 3ai

The points shown on the chart below represent monthly online sales in Australia.

The variable y represents sales in millions of dollars.

The variable t represents the month when the sales were made, where $t = 1$ corresponds to January 2021, $t = 2$ corresponds to February 2021 and so on.

a. A cubic polynomial $p : (0, 12] \rightarrow R, p(t) = at^3 + bt^2 + ct + d$ can be used to model monthly online sales in 2021.

The graph of $y = p(t)$ is shown as a dashed curve on the set of axes above.

It has a local minimum at (2, 2500) and a local maximum at (11, 4400).

i. Find, correct to two decimal places, the values of a, b, c and d.

Good use of standard algebra functionality

Define $p(t)=at^3+bt^2+ct+d$

done

$$\left\{\begin{array}{l} p(2)=2500 \\ p(11)=4400 \\ \frac{d}{dt}(p(t))=0 \mid t=2 \\ \frac{d}{dt}(p(t))=0 \mid t=11 \end{array}\right| a,b,c,d$$

{a=-5.212620027, b=101.6460905, c=-344.0329218, d=2823.182442}

$$\text{solve}\left(\left\{\begin{array}{l} p(2)=2500 \\ p(11)=4400 \\ \frac{d}{dt}(p(t))=0|t=2 \\ \frac{d}{dt}(p(t))=0|t=11 \end{array}\right., \{a,b,c,d\}\right)$$

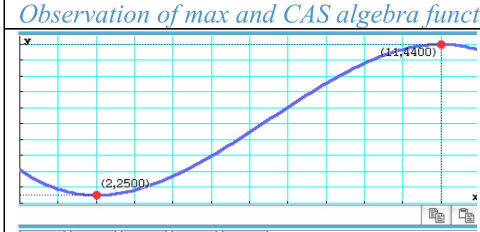
a=-5.21262002743 and b=101.646090535 and c=-344.032921811

Question 3aii

ii. Let $q : (12, 24] \rightarrow R, q(t) = p(t - h) + k$ be a cubic function obtained by translating p, which can be used to model monthly online sales in 2022.

Find the values of h and k such that the graph of $y = q(t)$ has a local maximum at (23, 4750).

Observation of max and CAS algebra functionality



Sheet1 Sheet2 Sheet3 Sheet4 Sheet5

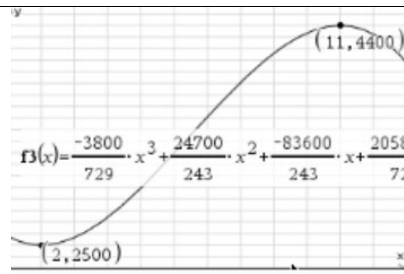
$y21=\frac{-3800\cdot x^3}{729}+\frac{24700\cdot x^2}{243}-\frac{83600\cdot x}{243}+\frac{2058100}{729}$

Define $q(t)=p(t-h)+k$

done

$$\left\{\begin{array}{l} q(23)=4750 \\ \frac{d}{dt}(q(t))=0 \mid t=23 \end{array}\right| h,k$$

{{h=12, k=350}, {h=21, k=2250}}



Question 3bi

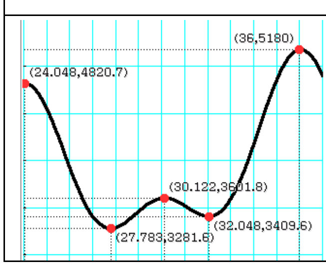
b. Another function f can be used to model monthly online sales, where

$$f:(0,\,36] \rightarrow R,\; f(t) = 3000 + 30t + 700\cos\left(\frac{\pi t}{6}\right) + 400\cos\left(\frac{\pi t}{3}\right)$$

Part of the graph of f is shown on the axes below.

i. Complete the graph of f on the set of axes above until December 2023, that is, for $t \in (24, 36]$.

Label the endpoint at $t = 36$ with its coordinates.



Question 3bii

ii. The function f predicts that every 12 months, monthly online sales increase by n million dollars.

Find the value of n.

Simple subtraction of peak sale values in previous image

Question 3biii

iii. Find the derivative $f'(t)$.

$$\text{simplify}\left(\frac{d}{dt}\left(3000+30\cdot t+700\cdot\cos\left(\frac{\pi\cdot t}{6}\right)+400\cdot\cos\left(\frac{\pi\cdot t}{3}\right)\right)\right)$$

$$\frac{-350\cdot\sin\left(\frac{t\cdot\pi}{6}\right)\cdot\pi}{3}-\frac{400\cdot\sin\left(\frac{t\cdot\pi}{3}\right)\cdot\pi}{3}+30$$

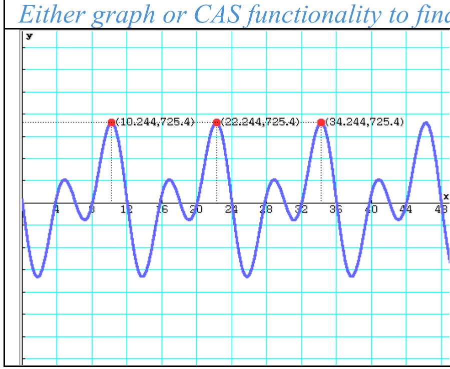
$$\frac{d}{dt}\left(3000+30\cdot t+700\cdot\cos\left(\frac{\pi\cdot t}{6}\right)+400\cdot\cos\left(\frac{\pi\cdot t}{3}\right)\right)$$

$$\frac{-400\cdot\pi\cdot\sin\left(\frac{\pi\cdot t}{3}\right)}{3}-\frac{350\cdot\pi\cdot\sin\left(\frac{\pi\cdot t}{6}\right)}{3}+30$$

Question 3biv

iv. Hence, find the maximum instantaneous rate of change for the function f, correct to the nearest million dollars per month, and the values of t in the interval (0, 36] when this maximum rate occurs, correct to one decimal place.

Either graph or CAS functionality to find maximum values.



$$\text{fMax}\left(\frac{-400\cdot\pi\cdot\sin\left(\frac{\pi\cdot t}{3}\right)}{3}-\frac{350\cdot\pi\cdot\sin\left(\frac{\pi\cdot t}{6}\right)}{3}+30,t,0,36\right)$$

$t=10.2436993092$ or $t=22.2436993092$ or $t=34.2436993092$

Question 4a

At an airport, luggage is weighed before it is checked in.

The mass of each piece of luggage, in kilograms, is modelled by a continuous random variable X, whose probability density function is

$$f(x)=\begin{cases}\frac{1}{67500}x^2(30-x) & 0\le x\le 30\\ 0 & \text{elsewhere}\end{cases}$$

A piece of luggage is labelled as heavy if its mass exceeds 23 kg.

a. Write a definite integral which gives the probability that a piece of luggage is labelled as heavy.

No calculation

Question 4bi

b. i. Find the mean of X.

Routine integral

$\int_0^{30}\frac{1}{67500}x^3(30-x)dx$ 18	$\int_0^{30}\left(\frac{1}{67500}\cdot x^3\cdot(30-x)\right)dx$ 18

Question 4bii

ii. Find the standard deviation of X.

Routine integral

$\sqrt{\int_0^{30}\frac{1}{67500}(x-18)^2x^2(30-x)dx}$ 6	$\sqrt{\int_0^{30}\left(\frac{1}{67500}\cdot x^4\cdot(30-x)\right)dx-\left(\int_0^{30}\left(\frac{1}{67500}\cdot x^3\cdot(30-x)\right)dx\right)^2}$ 6

Question 4biii

iii. Given that the mass of a piece of luggage is more than the mean, find the probability that it is labelled as heavy, correct to three decimal places.

Routine integral

$\frac{\int_{23}^{30}\frac{1}{67500}x^2(30-x)dx}{\int_{18}^{30}\frac{1}{67500}x^2(30-x)dx}$ 0.4457500565	$\frac{\int_{23}^{30}\left(\frac{1}{67500}\cdot x^2\cdot(30-x)\right)dx}{\int_{18}^{30}\left(\frac{1}{67500}\cdot x^2\cdot(30-x)\right)dx}$ 0.445750056459

Question 4c

c. Let W be the discrete random variable that represents the number of pieces of luggage labelled as **heavy** checked in by each traveller.

i. Show that $\Pr(W=2)=0.027$, correct to three decimal places.

Show that question; could have used binomial to get part of the solution, but without calculator syntax. Not CAS.

Question 4cii

ii. Complete the table below for the probability distribution W, correct to three decimal places.

Tree diagram worked best. Only CAS to multiply branches.

Question 4di

d. On a particular day, a random sample of 35 pieces of luggage was selected at the airport.

Let $\hat{P}$ be the random variable that represents the proportion of luggage labelled as heavy in random samples of 35.

i. Find $\Pr(\hat{P}>0.2)$, correct to three decimal places.

Routine cumulative binomial distribution after sample proportion conversion

binomialCDf(8, 35, 35, 0.234) 0.5952757011	binomCdf(35,0.234,8,35) 0.595275696505

Question 4dii

ii. Determine the probability that $\hat{P}$ lies within one standard deviation of its mean, correct to three decimal places. Do **not** use a normal approximation.

Sample proportion summary statistics needed first, then a routine cumulative binomial distribution.

$\sigma=\sqrt{\dfrac{0.234(1-0.234)}{35}}$ $\sigma=0.07156295929$ binomialCDf$(6,10,35,0.234)$ 0.6838823111	$\sigma=\sqrt{\dfrac{0.234\cdot(1-0.234)}{35}}$ $\sigma=0.071562959294$ binomCdf$(35,0.234,6,10)$ 0.683882310914

Question 4ei

e. i. In one random sample of 50 pieces of luggage, 10 are labelled as heavy.

Use this sample to find an approximate 90% confidence interval for p, the population proportion of luggage labelled as heavy, correct to three decimal places.

Statistical function of confidence intervals

Type: Interval One-Prop Z Int C-Level 0.9 x 10 n 50 Lower 0.106953 Upper 0.293047 $\hat{p}$ 0.2 n 50	**1-Prop z Interval** Successes, x: 10 n: 50 C Level: 0.9 zInterval_1Prop 10,50,0.9: *stat.results* "Title" "1-Prop z Interval" "CLower" 0.106953027765 "CUpper" 0.293046972235 "$\hat{p}$" 0.2 "ME" 0.093046972235 "n" 50.

Question 4eii

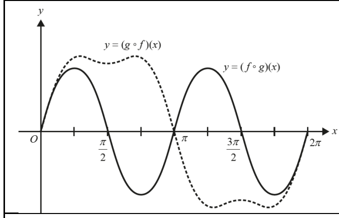
ii. A second random sample of 50 pieces of luggage is selected. Using this sample, the approximate 90% confidence interval for p, the population proportion of luggage labelled as heavy, is **wider** than the one obtained above in **part e.i**.

State the minimum and maximum possible number of pieces of luggage labelled as heavy in the second sample.

Construction of inequality was the difficult part. Good CAS use to solve the inequality

Long version $\text{solve}\left(\sqrt{\dfrac{\frac{n}{50}\cdot\left(1-\frac{n}{50}\right)}{50}}>\sqrt{\dfrac{0.2\cdot(1-0.2)}{50}},n\right)$ $\{10<n<40\}$ Short version $\text{solve}\left(\frac{n}{50}\cdot\left(1-\frac{n}{50}\right)>0.16,n\right)$ $\{10<n<40\}$	Long version $\text{solve}\left(\sqrt{\dfrac{\frac{n}{50}\cdot\left(1-\frac{n}{50}\right)}{50}}>\sqrt{\dfrac{0.2\cdot(1-0.2)}{50}},n\right)$ $10.<n<40.$ Short version $\text{solve}\left(\frac{n}{50}\cdot\left(1-\frac{n}{50}\right)>0.2\cdot(1-0.2),n\right)$ $10.<n<40.$

Question 5ai



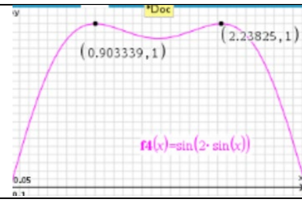
a. **i.** The graph of $y = (g \circ f)(x)$ has a local maximum whose x-value lies in the interval $\left[0, \frac{\pi}{2}\right]$.

Find the coordinates of this local maximum, correct to one decimal place.

CAS functionality of graph

$\text{fMax}\left(\sin(2\cdot\sin(x)), x, 0, \frac{\pi}{2}\right)$

$\{\text{MaxValue}=1, x=0.9033391108\}$



Question 5aii

ii. State the range of $g \circ f$ where $x \in [0, 2\pi]$.

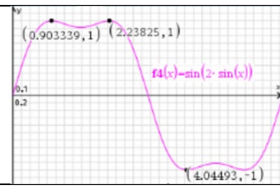
CAS functionality of graph (extension of last part)

$\text{fMin}(\sin(2\cdot\sin(x)), x, 0, 2\cdot\pi)$

$\left\{\text{MinValue}=-1, x=\sin^{-1}\left(\frac{\pi}{4}\right)+\pi, x=-\sin^{-1}\left(\frac{\pi}{4}\right)+2\cdot\pi\right\}$

$\text{fMax}(\sin(2\cdot\sin(x)), x, 0, 2\cdot\pi)$

$\left\{\text{MaxValue}=1, x=\sin^{-1}\left(\frac{\pi}{4}\right), x=-\sin^{-1}\left(\frac{\pi}{4}\right)+\pi\right\}$



Question 5bi

b. **i.** Find the derivative of $f \circ g$.

$\frac{d}{dx}(\sin(\sin(2\cdot x)))$

$2\cdot\cos(\sin(2\cdot x))\cdot\cos(2\cdot x)$

$\frac{d}{dx}(\sin(\sin(2\cdot x)))$ $\quad 2\cdot\cos(2\cdot x)\cdot\cos(\sin(2\cdot x))$

Question 5bii

ii. Show that the equation $\cos(\sin(2x)) = 0$ has no real solutions

Show that question. No critical CAS use

Question 5biii

iii. Find the x-values of the stationary points of $f \circ g$ where $x \in [0, 2\pi]$.

$\text{solve}\left(\frac{d}{dx}(\sin(\sin(2\cdot x)))=0, x\right) \mid 0\le x\le 2\pi$

$\left\{x=\frac{\pi}{4}, x=\frac{3\cdot\pi}{4}, x=\frac{5\cdot\pi}{4}, x=\frac{7\cdot\pi}{4}\right\}$

$\text{solve}(2\cdot\cos(2\cdot x)=0, x) \mid 0<x<2\cdot\pi$

$x=\frac{\pi}{4}$ or $x=\frac{3\cdot\pi}{4}$ or $x=\frac{5\cdot\pi}{4}$ or $x=\frac{7\cdot\pi}{4}$

Question 5

iv. Find the range of $f \circ g$ where $x \in [0, 2\pi]$

$\text{fMin}(\sin(\sin(2x)), x, 0, 2\cdot\pi)$

$\left\{\text{MinValue}=-\sin(1), x=\frac{3\cdot\pi}{4}, x=\frac{7\cdot\pi}{4}\right\}$

$\text{fMax}(\sin(\sin(2x)), x, 0, 2\cdot\pi)$

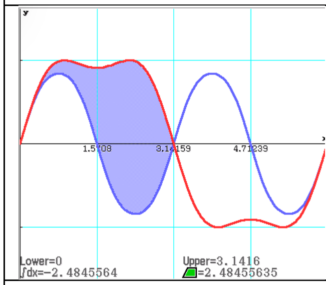
$\left\{\text{MaxValue}=\sin(1), x=\frac{\pi}{4}, x=\frac{5\cdot\pi}{4}\right\}$

$\sin(\sin(2\cdot x)) \mid x=\frac{\pi}{4}$ $\quad \sin(1)$

$\sin(\sin(2\cdot x)) \mid x=\frac{3\cdot\pi}{4}$ $\quad -\sin(1)$

Question 5ci

c. **i.** Write a single definite integral that gives the area bounded by the graphs of $y = (f \circ g)(x)$ and $y = (g \circ f)(x)$ in the interval $[0, 2\pi]$.



$$2\cdot\int_0^{\pi}\left(\sin(2\cdot\sin(x))-\sin(\sin(2\cdot x))\right)dx$$

4.96911270329

Question 5d

d. Let $f_1 : (0, 2\pi) \to R,\ f_1(x) = \sin(x)$.

Find all values of x in the interval $(0, 2\pi)$ for which the composition $f_1 \circ g$ is defined.

solve(sin(2·x)≥0, x)

$$\left\{\pi\cdot\text{constn}(1)\le x\le\pi\cdot\text{constn}(1)+\frac{\pi}{2}\right\}$$